Deliverable D3.3

Second Release of LA/EDM Services and Algorithms

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EXECUTIVE SUMMARY

Work package 3 is mainly concerned with research and technical development of software components in the field of CbKST and FCA. These components include functions for (i) collecting, (ii) accumulating, (iii) analysing, and (iv) interpreting educationally- relevant data ranging from conventional test results to broader activity data. Concrete functions cover

- evidence-based establishing and validating the teachers' domain models and teaching plans
- identifying individual learning paths and individual learning progress
- predicting individual learning trajectories
- adaptive assessments of competencies and competence states
- identifying individual learning styles
- evaluating the effectiveness of teaching methods and materials
- visualizing data and the results of analyses
- appropriate communication and reporting of teaching/learning activities
- appropriate communication and negotiation of individual learning achievements

In year two, our work primarily addressed aspects of the FCA-based analysis and the related software tools, in addition we focussed on the *Learning Performance Vector* and the *Learning Horizon*, which are predictive approaches on the basis of competence structures. Equally important was the wok on the visualization service of Hasse diagram-based reports. Finally, we attempted to come up with teacher-centred intuitive tools for teachers and students.

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1. CBKST and FCA Research

Learning analytics and educational data mining are two highly interrelated research fields which became enormously popular in recent years (e.g. Steiner, Kickmeier-Rust & Albert, 2014). When applying learning analytics and educational data mining in schools, it is of high importance to meet the requirements of teachers and students. Teachers usually want to have user-friendly tools which help them to reduce the time required for personalized assessment and tailored competence development of their students. LEA´s BOX extends existing frameworks: the *Knowledge Space Theory* (KST) and the *Formal Concept Analysis* (FCA). These two frameworks are well established in the fields of student modelling (KST) and domain modelling (FCA) - based on order- and lattice theory. They serve as theoretical basis for structuring, analyzing and visualizing educational data.

1.1. Knowledge Space Theory

The KST (Doignon and Falmagne, 1985) suggests that every knowledge domain *Q* (e.g. descriptive statistics) can be characterized by a set of problems (items). A student´s knowledge state is the set of problems he or she is able to master. In many cases, it is reasonable to assume mutual dependencies, so-called prerequisite relations, between the problems of a given knowledge domain. For example, a student who successfully masters problem *y* (e.g. calculation of standard deviation) presumably masters problem *x* (e.g. calculation of means) too. In this case, problem *x* is a prerequisite of problem *y*. A *knowledge space* is the ordered set of all reasonable knowledge states. Reasonable in this context means, that a knowledge state which includes a particular problem also includes the problem´s prerequisites (in the example above, all knowledge states which include problem *y* also include problem *x*). A knowledge space also includes the empty set (a student may not master any problems) as well as the set *Q*. For additional properties of knowledge spaces see Doignon and Falmagne (1999).

The KST has a 30-years tradition as powerful framework for learner modelling, adaptive testing and competence development in technology-enhanced learning (for an overview see Falmagne et al., 2013), and thus, the main focus of this paper is on the FCA which hasn't´ been extensively applied for such kind of purposes so far.

1.2. Formal Concept Analysis

The *F*ormal *C*oncept *A*nalysis (FCA) has been established in the early 80s by Wille and colleagues (Wille 1982, 2005). The FCA aims to describe a domain, i.e. concepts and concept hierarchies in mathematical terms. The starting point is the definition of the formal context. A formal context K is defined as a triple (*G*, *M*, *I*) with *G* as a set of objects (in German: "Gegenstände") and *M* as a set of attributes *M* (in German: "Merkmale"). The relation *I* (incidence-relation) assigns objects and attributes, i.e. *g I m* means the object g has the attribute m. The formal context K can be represented as a cross table, with the objects in the rows, the attributes in the columns and by crosses ("Xs") whenever *g I m* holds for a particular object and attribute (see table 1).

Table 1: Example of a formal context with objects and their attributes

Objects *G*	is toxic	is able to fly	is able to swim	hattched from egg
Bee	X	X		X
Bumble-bee		X		X
Tree frog			X	X
Grass snake	X		X	X

For each subset $A \subseteq G$ and $B \subseteq M$ the following derivation operators need to be defined:

$A \rightarrow A'$:= {m $\in$ *M*| g*I*m for all g $\in$ *A*} which is the set of common attributes of the objects in *A*, and
$B \rightarrow B'$:= {g $\in$ *G*| g*I*m for all m $\in$ *B*} which is the set of objects which have all attributes in *B.*

A formal concept is a pair (*A*, *B*) with the subsets $A \in G$ and $B \in M$ which fulfil $A = B'$ and $B' = A$. The set *A* is called the *extension* of the formal concept; it is the set of objects of the formal concept. The set *B* is called the *intension* of the formal concept; it is the set of attributes which apply to all objects of the extension. The ordered set of all formal concepts is called the concept lattice *B*(K) (see Wille, 2005) which can be visualized by a labelled line diagram (see Figure 1).

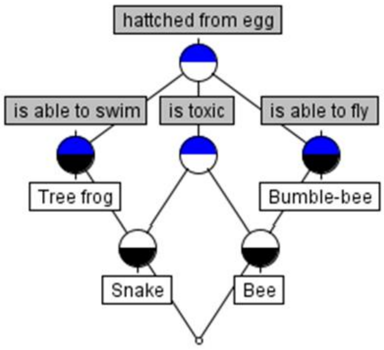


Figure 1. A concept lattice resulting from the formal context in table 1

Every node represents a formal concept. In order to avoid redundancy, all objects and attributes are labelled only once. A concept lattice can be "read" as follows: The extension *A* of a formal concept comprises all objects whose labels can be reached by descending paths. As an example, the node with the label "Tree frog" has the extension {Tree frog, Snake}. The intension *B* of a formal concept can be reached by all attributes whose labels can be reached by ascending paths from that node. In the case of the formal concept in the example above, the intension consists of the attributes {hatched from egg, is able to swim}.

1.3. Applying the FCA for Learner Modelling

Rusch and Wille (1996) were the first who applied the FCA with learners and their knowledge states to show the correspondence between the FCA and the KST. They proposed a knowledge context (*S*, *P, I)* with students *S*, problems *P* and an incidence-relation which assigns students to problems which they have *not* solved. This rather unintuitive incidence relation leads to formal concepts whose complements of the intensions are knowledge states.

However, for LEA´s BOX such kind of knowledge contexts or concept lattices are not applicable since it is not intuitive for teachers to think in terms of "complements of a formal concept´s intension". They are mainly interested in clear visualizations which directly indicate the set of problems which have been mastered by a student (or which they failed).

We suggest knowledge contexts with student as "attributes" and problems as "objects". An example of such a knowledge context is given in table 2 (the data has been reported by Korrossy, 1999). Such an alternate knowledge context overcomes the above mentioned shortcut since a student´s knowledge state can be directly derived from the according concept´s extension. In addition to that, as it will be outlined in the following sections, the resulting concept lattice allows visualizing answers to a set of pedagogical questions which might be of interest for teachers.

Table 2: A knowledge context with student as attributes and problems as objects (from Korossy, 1999)

Problem	*01*	*02*	*03*	*04*	*05*	*06*	*07*	*08*	*09*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*
a	X	X	X	X	X	X		X	X		X	X	X	X	X	X	X	X	X	X	X	X	X
b		X	X	X	X		X		X		X	X		X	X		X			X	X		
c	X	X	X		X	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
d	X		X		X		X										X						
e	X	X	X		X		X				X	X	X				X						X
f	X	X	X				X						X		X		X						X

Depicting Knowledge States from Formal Concepts Extensions

The concept lattice which results from the knowledge context in table 2 is shown in figure 2. As briefly outlined above, the set of problems which have been solved by a particular learner can be directly depicted from the extension of the formal concept with the learners´ label assigned to it. As an example in figure 2 (left side), the student *04* has successfully mastered the problems *a* and *b*. Student *10* is the only one who solved only a single problem, *c*, and students *03* and *17* (assigned to the top element of the concept lattice) mastered all problems.

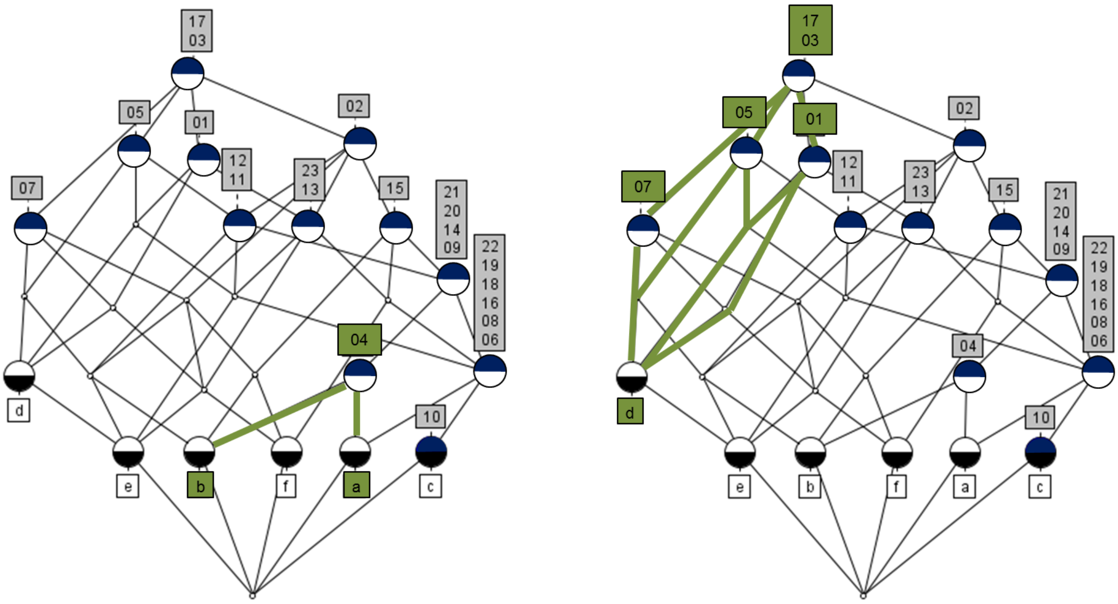


Figure 2. The extension of a formal concept is a knowledge state (left side) and the intensions of a formal concept with an problem-label is the set of students who solved that problem (right side)

Depicting the set of students who solved items from formal concepts intensions

The intension of a formal concept which has an problem-label assigned to it indicates the set of students which have succesfully mastered that problem. As an example, the problem *d* in figure 2 (right side) has been solved by the learners *01*, *03*, 05, *07* and *17*. As it can be also seen, this formal concept located above the formal concept with the problem-label *e* assigned to it. This means, that all students who solved problem *d* were also able to solve problem *e*, i.e problem *e* can be considered as prerequisite for problem *d*.

Highlighting overlaps and differences of students performances

The performances of two or more students can be compared when examining the intensions of the formal concepts with the according attribute-labels. As exemplified in figure 3, the students *07* and *15* mastered different subsets of problems. The knowledge state of student *07* encompasses the problems solved *b*, *d*, *e* and *f* while the knowledge state of student *15* encompasses the problems *a*, *b*, *c*, and *f*. Both students mastered problems *b* and *f* (which is the *set closure* of their intensions) and together they mastered all problems (which is the *set union* of their intensions).

As a teacher, such kind of information might be of great interest since it helps to effectively arrange groups of students when aiming for collaborative, peer-learning (where students learn together in groups). In the example above, the students *07* and *15* together could be tutors for other students.

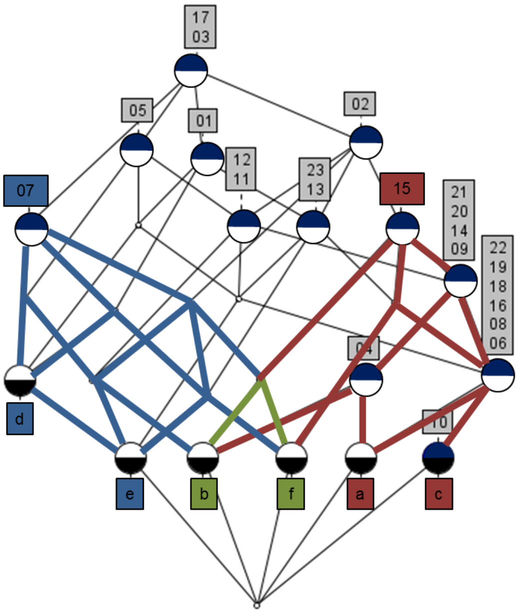


Figure 3. Comparing performance patterns between subsets of learners

Visualizing a classrooms´ learning progress over time

The concept lattice in figures 2 and 3 results from a formal context which is an evaluation of the students´ performances at a certain point in time. However, in some cases it might be of great interest for a teacher to observe the learning progress over a longer period of time. In the perfect case, all students should finally end up (e.g. at the end of the semester) with the knowledge state *Q*. In such a case, all cells in the knowledge context would be filled with crosses. This would result in a concept lattice with only a single formal concept. Figure 4 exemplifies such an ideal learning progress. The concept lattice in the middle results from adding one solved item to the students´ knowledge states (except for the students *03* and *17*). The concept lattice on the right side results from adding another item to all knowledge states smaller than *Q*.

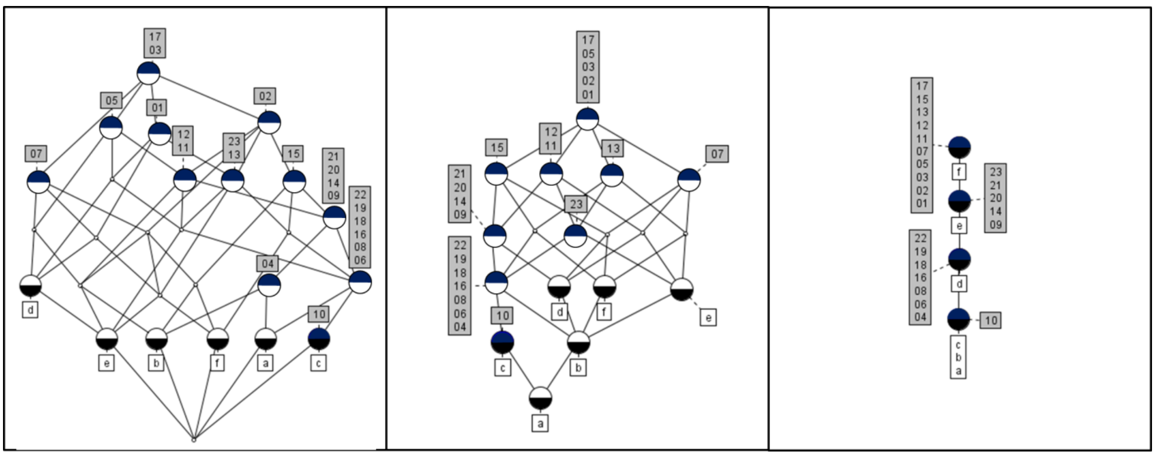


Figure 4. Changing concept lattices over time reflects learning progress of the whole class of students

Such a kind of interactive visualization (which could be manipulated for example with a slider) might be of particular interest when dealing with competences rather than on a rather behaviorist performance level (i.e. solved or failed problems; see for example competence-based extensions of the KST, e.g. Albert & Lukas, 1999; Heller, Steiner, Hockemeyer, & Albert, 2006). In general, the visual appearance of the concept lattice gives an first impression of the students coherence: A concept lattice which looks "complex" due to a large amount of formal concepts is an indication for a high diversity among the students´ performance- and competence states. On the other side, a concept lattice with a relatively small amount of formal concepts indicates that the students with respect to the knowledge or competence states are more coherent.

1.4. Answering pedagogical questions with FCA

In addition to the conceptual work we defined a set of important pedagogical questions for Learning Analytics. The conceptual research in the context of FCA aimed at finding solutions to provide answers to the pedagogical questions on the basis of the data in the Lea's Box system. We summarized these questions and the solutions already in the system design document 2 (D2.2). In the context of the project, we translated the demands on FCA into a fully integrated FCA tool (see the system release deliverable D2.5 for detailed explanations and the manual for the tool).

2. Learning Performance Vector – Learning Horizon

2.1 Introduction

In the center of conceptual research in the field of CbKST and FCA was the so called Learning Performance Vector (LPV) and the Learning Horizon. The principle idea of this constructs is to use CbKST and FCA as means of predictive analytics. The fundamental idea, thereby, is to consider the past learning performance in terms of CbKST-like learning paths , the current progress of an individual learner as well as a summary of peer performance (if available) and to match learning time and remaining time with the learning goals. In such a way we aim at deriving estimations of an individual's learning success and the degree to which a desired learning goal can be achieved. The foundations of this approach are not only competence structures and formal concepts (e.g., competencies over learners) but also temporal information, weighting information of activities and achievements, and difficulty aspects of future learning tasks. In the end, we try to establish an algorithm that is capable of melding those information into robust predictions of learning success – in other terms of the likelihood the a particular student can reach the learning goals in a given amount of time – the Learning Horizon. Of course, the predictions are unstable and blurred in the beginning and certainly the predications are more valid, the more time has passed and the more information the system has. Still, the approach is capable, so we hope, to give early indications of performance problems, so that it is still possible for educators to intervene appropriately. In addition, a particular strength is that the CbKST/FAC approach allows for finding concrete directions where a learner needs support and guidance.

2.2. Elements of the Learning Horizon and the LPV

2.2.1. Competence Structures and Performance

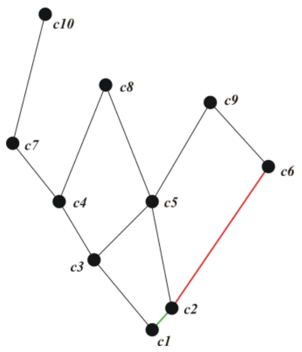
The first element we consider is clearly a competence structure. Very briefly, we decompose a learning domain (e.g., 2nd grade maths) into atomic chunks of knowledge or aptitude. In a second step we try to find a natural course of learning or, in other terms, we try to find the prerequisite structure: which elements need to be learned before another piece can be acquired. This gives us a combinatorics model of a learning domain and a certain understanding of how learning and development occurs. Now, it must be highlighted that competencies and learning, abilities and aptitudes are latent constructs. One cannot directly observe the real "knowledge" of another person. It takes indicators and evidences, in its simplest form a school test. We know, very well, that tests are not necessarily objective. Student's be inattentive and fail although the have the knowledge or competence, some may guess the right answer incidentally. So in the end, there is a good portion of uncertainty in assessment. When talking about the underlying competencies, we need to account for this fact. And we need to account for that in a careful and conservative way. The CbKST approach des that by establishing stochastic relationships. Each indicator, each piece of evidence, each test result is only one indicator that contributes to the whole picture, but it contributes only with a certain probability. The more evidence we can aggregate, mirroring the same competencies and competence structures, the clearer and more robust our picture (our model of the learner) gets. Of course, we have to consider that different evidences have different weights, a different impact, on the learner model. A simple multiple choice test weighs less than an oral exam within which a teacher can explore the real knowledge of a student, exhibiting abilities in real live weighs more than filling in the right answers. The conceptual details of CbKST, which has a long tradition in intelligent, adaptive tutorial systems, are given in the previous deliverables (particularly D3.1).

2.2.2. Formal Contexts

FCA, the analysis of formal context, is a related formal psychological approach. The idea is to identify patterns in a universe of two dimensions. Imagine there is a set of competencies and a set of students. There is a multitude of clusters, some students hold the one some the other competencies. FCA allows to quickly analyse the patterns and identify relevant clusters, even more, hierarchies. If FCA is applied on the competency models of CbKST, we have the opportunity to meld pedagogically inspired domain models with pattern identification mechanisms. By this means we can identify clusters of good and not so good learners, we can establish a hierarchy of performance, and, at each step, we can determine which competencies are lacking, and therefore which educational measures would be necessary. As outlined in D2.2, there is a broad variety of educationally relevant questions that can be addressed using the paired CbKST / FCA approach.

2.2.3. Likelihoods, Weights, and their Extensions

In recent works we demonstrated that the traditional approaches of using Hasse diagrams for visualizing competence structures and lattice graphs for displaying formal contexts can be extended in meaningful ways. One idea suggested by (Kickmeier-Rust, Steiner, & Albert, 2015) was to extend Hasse diagram visualizations by adding a difficulty (a weight) dimension to the diagram by making edges correspondingly longer. There are two important aspects to this idea. On the one hand, it introduces weights, levels of difficulties, efforts to make the step from one to another competence state, on the other hand, it provides valuable information to inspire the LPV and the estimation of a Learning Horizon. He following figure gives a conceptual indication about how different weights/difficulties/required efforts might influence a Hasse diagram visualization and, therefore, the conceptual analytics algorithm. In addition to that, a simple yet important fact is that subject matter is increasing in difficulty over time. This definitely must be another variable in our model of learning.



Illustrating learning efforts (as costs or pace). The longer the more efforts/time it took to acquire a further competency.

2.2.4. What peers are doing

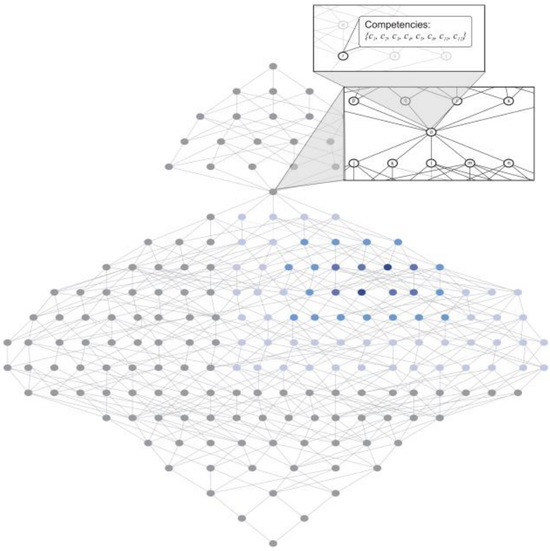
Now, when it's about to estimate a student's potential progress and chances to accomplish a course on time, e central element is a comparison to other learners. [It shall be highlighted that this is optional, since the LPV can be computed without peer information!] If a particular student appears being clearly ahead of the majority or, in a worse case, behind the majority, a teacher can receive corresponding and actionable information from analytics.

Here also a meta-perspective comes into play, namely the degree to which a teacher is capable of setting the right learning goals for a particular group of students and the ability to reach the goals. This is a non-trivial aspect to Learning Analytics tools. Oftentimes, a teacher is seen as the ultimate key luminary in a certain domain. This, however, is not necessarily true. Teacher may completely misjudge the abilities and potentials of a group of students (and there is a variety of reasons why this may happen). So, a dimension of a group comparison can add substantial information about individual progress as well as a teacher's plans. In the end, this analysis offers a fountain of deeper insights.

Finally, it's worth mentioning that a theoretically sound peer comparison offers the option for a motivation boost of individual efforts, almost like the principle of badging or gamification. Position and achievements in peer groups have tremendous motivational powers, however, the must be utilized very carefully and thoughtfully!

2.2.5. The Algorithm

To summarize all that it takes an algorithm, or in other terms, mathematical statements. So what do we have: A competence structure (or competence space(next figure)).

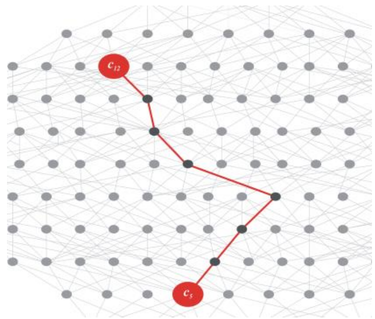


This structure gives us a model of the learning domain, starting from point 0 (in this particular domain) leading to the complete mastery. In other terms, a competence structures is the manifestation of all possible and reasonable states a person can be in. This allows us to identify the progress of a particular learner given the timeline of a course. Mathematically speaking we have the sum of all possible learning paths. This indicates the average learning efforts, given that transitions have specific difficulties or weights (as explained above).

We have a set of competencies *Q = {a, b, c, …. }* with a relationship $c \geq c'$ among the competencies, which establishes the competence structure. The sum of the resulting competence states is $\Sigma(|Q|r)$. Given that the transitions from one competence state to another has a difficulty parameter, which in turn is the average of the difficulty parameters of the competencies being a part of the state, we have a set of tuples of the start competence state, the end state, and the difficulty τ = *[s1, s2, w]*. This results in a set of such tuples for the entire competence structure $T = \Sigma(\tau|Q)$. Also, we have a set of indicators providing evidences for competencies: $I = \{ e^i, \{c\} * w\}$, with a given weight *w*.

Based on the evidences we can estimate the likelihood of each competency. The probability of a competence state is the average of its competencies $\Pi(s) = \Sigma(\pi)/n$.

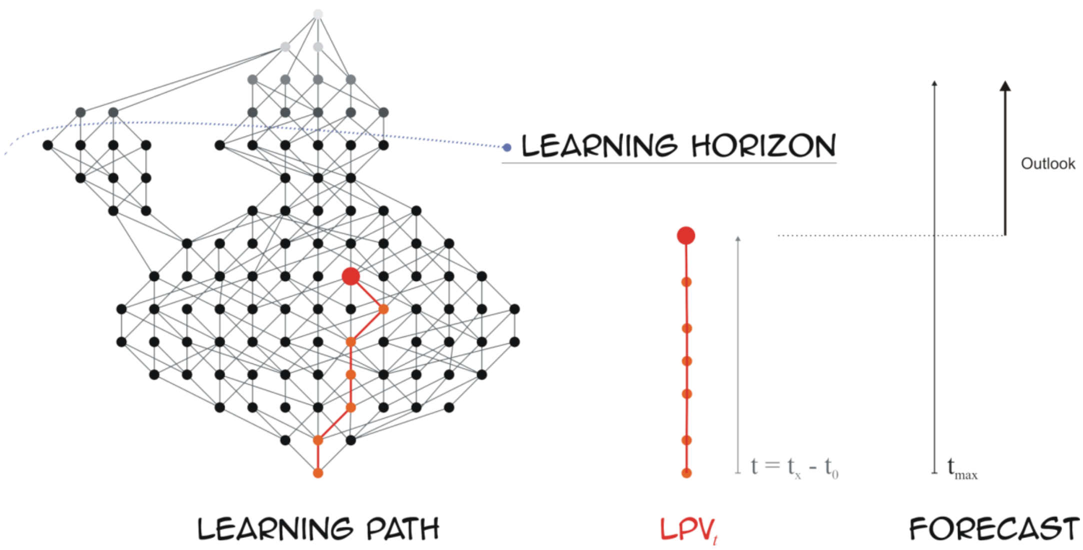
To identify the learning path of a person, we identify the state with the highest probability in certain time steps. Depending on the nature of the concrete use case this may rely on the events when evidences are put into the system or, alternatively on a timely basis (e.g., weekly or monthly). This is basically illustrated in the next figure.



Learning Path. The cutout is part of the structure shown above.

Now for each step we compute the difficulty (as a value from 0 to 1). The sum of the values gives us an indicator for how many efforts a student has to spend on her learning history (the individual learning path). In a next step, given the concrete competence state of the learner, we have to identify the possible paths towards to defined learning goal, which is a (rather small) subset of all possible paths. Equally to the computation of the difficulty to reach the current state, we can compute the potential difficulty of all possible paths to the goal, whereas we have to compute the average difficulty of all possible paths. This now is an indicator for the efforts that are necessary for an individual learner to reach the learning goal.

As indicated in the following figure, when link the progress of a student within a given span of time, we can make a prediction about how far a student can come within the remaining time (of a course, for example). So, as a final step, we can identify exactly those states (and therefore the competencies) a particular will be able to reach within the time limits. The set of those states is, now finally, the student's Learning Horizon.



2.2.6. Current Status and Outlook

Since the LPV and the Learning Horizon approach have not the highest priority in the project, the current state of the research and implementation activities is at an early stage. We are still working on finding the right algorithms for this computationally highly demanding approach. In addition we are conducting simulation studies aiming at identifying the boundaries for such approach. This means the number of competences and competency states is a critical factor for a computation in real time (although real time computability is not necessarily a crucial factor for such kind of predictions). In addition, we try to use the available large scale data from Turkish systems (Vitamin) and the US systems (Adaptive Curriculum) to start validation studies of the approach.

We expect to have a first demonstrator version of the approach ready for the final release of the Lea's Box system, along with some simulation studies on the basis of the 'real' data.

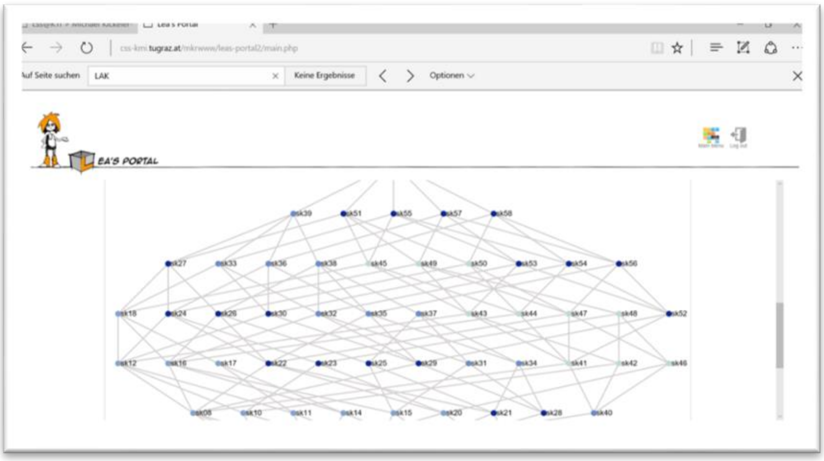
3. Hasse Diagram Visualization

Another part of the work in year two concerned still the developed of web-based displays of learning paths and competence states on the basis of structured graphs, in particular Hasse diagrams.

A Hasse diagram is a mathematical representation of a so-called semi-order which helps for structuring learning domains and for visualizing the progress of a learner through this domain. The properties of a semi-order are: (i) reflexivity, (ii) anti symmetry, and (iii) transitivity. The representation of this diagram is illustrated in the image below. The direction of a graph reads from bottom to top. The arrows from one element to itself (reflexivity property) as well as all arrows indicating transitivity are not shown, but they are included (used) so far.

In an educational context, a Hasse diagram can display the non-linear path through a learning domain starting from an origin at the beginning of an educational episode (which may be a single school, lesson or the entire semester). The beginning is shown as a {0} (empty set) at the bottom of the diagram. Now a learner might focus on three topics (X, Y, Z). In essence this establishes three possible learning paths, until reaching the final state (X, Y, Z).

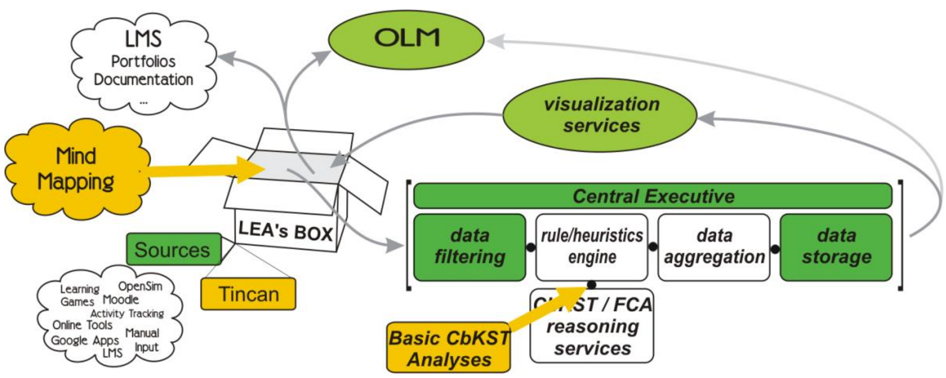
In the context of formative learning analytics, a competence-oriented approach is necessary. Thus, a Hasse diagram can be used to display the competencies of a learner in the form of so-called competence states. The knots of this Hasse diagram indicate meaningful competence states of a student while the edges indicate admissible transitions from one competence state to another by acquiring another competency. In addition, the approach is based on a probabilistic view of having or lacking certain competencies. Very briefly, a Hasse diagram shows all possible (admissible) competence or knowledge states. The visualization in the form of Hasse diagrams, finally, allows identifying the learning paths, the history of learning, the present state, and – most importantly, to find proper recommendations for the next and the very next learning steps. In Year two we accomplished significant advancements of the Hasse diagram visualization feature for Learning Analytics and integrated them as an integral part of the Lea's Box system.



Y2 implementation and integration of the Hasse visualization tool.

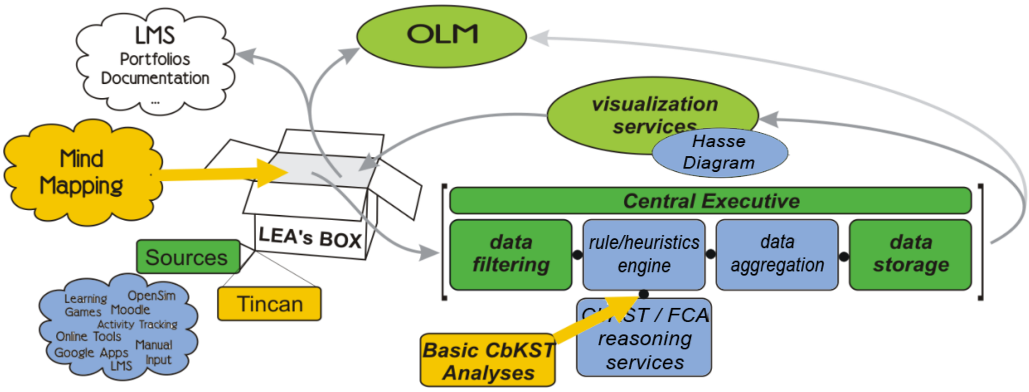
4. TECHNICAL IMPLEMENTATION

By the original architecture, the principle idea was to have the web platform (the box) that is equipped with an open interface to existing sources (i.e., tools, websites, apps, that are producing educationally relevant data). This data subsequently is processed within the platform and finally fed back to the user. The central component is a mechanism that controls the data flow and the deeper processing. In year one, we released the components marked in green in the figure above. Further we extended this setup by additional components (marked in yellow in the following figure). First we integrated basic CbKST-based function to identify competence states on the basis of performance data. This gives the first system release a base functionality for analyses. Second, in addition to the main API of the system (LEA'S API), we integrated a Tincan based interface to enable a broader connectivity and interoperability. Third, we developed and released a major tool for competence and domain modelling, the so-called mind mapping tool. The tools are tailored to teacher's needs (based on the focus group and design studies of WP5).



Extended architecture; new elements are shown in yellow color.

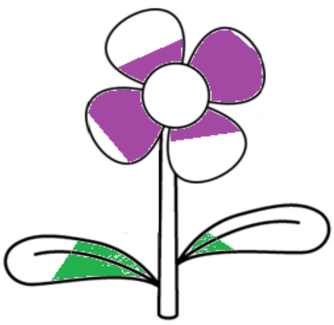
In year 2 we completed the envisaged developments (see the blue section in the following figure. In particular, we added the CbKST heuristic interpretation engine, we have a solid data aggregation feature to accommodates the needs of all system components (in particular myClass and the OLM). Also we established a robust data architecture and a functional API for external tools.

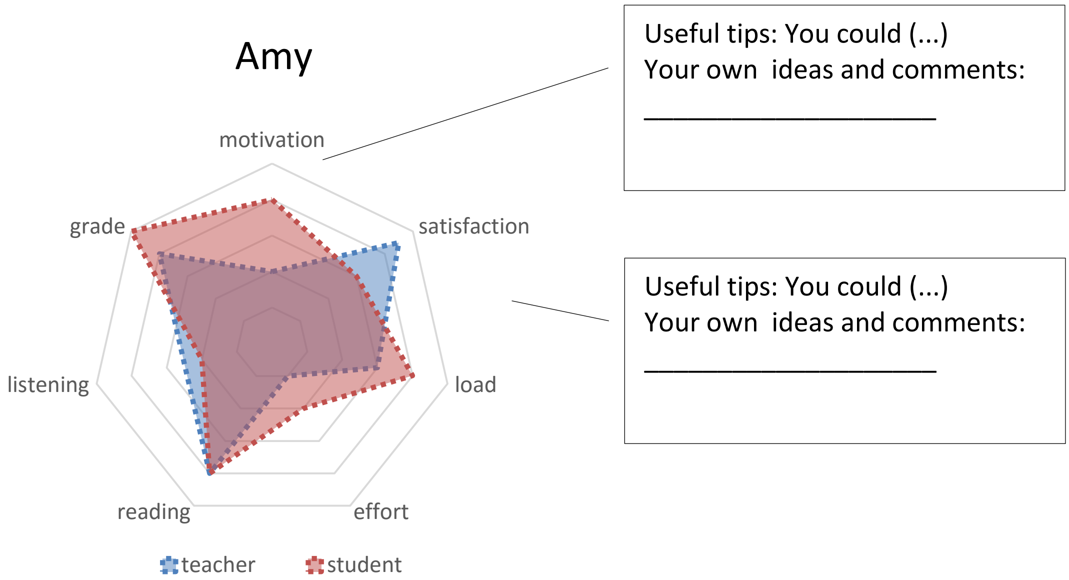


Extended architecture; new elements are shown in blue color.

5. Serving Teachers' Needs

As emphasized already, serving the very concrete needs and context conditions of teachers is a must to establish collaboration with real life educational scenarios. A specific sub-project worth mentioning is the investigation of 'different', innovative types of visualizations of competencies, competence states and learning progress, perhaps also the learning horizon. On approach we will be investigating in Y3 is the flower metaphor. This solution is a direct proposal from close interactions with teachers. The flower depicts the learning domain, the leaves and petals represent the topics, competencies and sub-competencies of a learning domain. The achievements (the learning vector) are depicted in form of the degree to which leaves and petals are filled with colour. Clicking on the parts of the flower allows drilling deeper into the learning history and to provide suitable recommendations (see the very next figure).





6. Outlook

In the remaining period our main focus will be on making the system stable and robust. This is a key requirement to produce impact and to deliver a durable project result. However, exactly this task is usually the most time consuming in a development process. Not least this is the reason why a good portion of school studies were conducted using isolated sub components of the system. When working with 'real' schools, it's inevitable that we need to provide them with robust and secure software tools. This cannot be done with leading edge research prototypes. Thus, a big portion of tools for schools are based on the heritage of prior projects. In Lea's Box we managed to make them mature enough to find their ways into school reality. The aim for the remaining months is to deliver a system (perhaps not including all the sophisticated research demonstrators) that is stable and reliable.

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